# Solutions to current crowding in circular vias for contact resistance measurements

L. H. Allen and M. Y. Zhang
*Materials Science Department, CSL 1101 West Springfield Street, University of Illinois, Urbana–Champaign, Urbana, Illinois 61801*

J. W. Mayer
*Material Science & Engineering Department, Bard Hall, Cornell University, Ithaca, New York 14850*

E. G. Colgan and R. Young
*IBM General Technology Division, Hopewell Junction, New York 12533*

(Received 14 September 1989; accepted for publication 25 March 1991)

The effects of current crowding near circular contacts has been analyzed. We analyze a simple system of two parallel plates connected by a cylindrical plug. Under a given set of assumptions the problem can be reduced from three-dimensional to one-dimensional geometry. Given this assumption, analytic solutions are obtained for the current and voltage distributions within the plug. From these expressions the correct values for contact resistivity ($P_c$) are derived. Finally, the analytical expressions are compared with the results from two-dimensional numerical calculations.

## I. INTRODUCTION

One of the basic elements in integrated circuit (IC) technology is making electrical connections between the different components of the circuit. Interest in electrical contacts[2] is of importance because their characteristics may limit the performance of the device.

In order to quantify the effects of the contact resistance on device performance, a detailed distribution of current and voltage near contact regions is required. By using this information we can obtain a value for the contact resistance, which is the characteristic parameter used in designing and analyzing the devices.

The most frequently cited analysis of contact resistance was given by Berger[1] in 1972. He analyzed the one-dimensional problem of two (top and bottom) parallel plates partially joined together at overlapping edges. In this case the current distribution is determined when current is supplied to the extrema of the plates. Current is transferred from one plate to the other over a given distance referred to as the *transfer-distance* $L_T$. In practice, one measures the total resistance of the contact, then calculates the value for $L_T$. Given the values of $L_T$, the interface contact resistivity $P_c$ can then be obtained.

In this paper we investigate a similar type problem, but one which has two-dimensional geometry. This configuration consists of two parallel plates (top and bottom) joined in the middle by a short cylinder plug. When current is applied to the outer perimeter of the plates it advances through the plate radially (laterally) inward toward the via. Current is then transferred through the plug region and then proceeds radially outward away from the via through the lower plate.

In certain cases the current density in the cylinder is uniform and the analysis is trivial. But in the case where current crowding occurs, the current density is nonuniform and is localized near the perimeter region of the plug. Our goal is to determine the path of the current through the plug for both uniform current density and current crowding conditions.

In modeling this problem we include the following aspects: the dimensions of the plug, the effects of the sheet resistance of the two parallel plates and the two interface resistances between the plates and plug. From this we determine the spatial distributions of the current and obtain the effective contact resistance of the total system. The problem is examined over an extended range of the parameters, however the model is restricted by certain assumptions which will be defined within the analysis.

The current distributions within the plug are also obtained using computer aided numerical methods. Both analytical and numerical methods are compared over a wide range of conditions.

## II. CURRENT CROWDING AT CIRCULAR VIAS

A schematic diagram of the general structure is shown in Fig. 1. It consists of two parallel horizontal plates joined at the center by a short cylindrical plug. There are two contact resistivities ($\Omega\,\mathrm{cm}^2$) which must be considered. They are attributed to the thin interface regions at the two plug/film interfaces. Current proceeds from the outer regions of the top thin film toward the via, then is transferred to the cylinder.

Under certain conditions, which will be discussed in the following section, the vertical current density (at the interface) will be uniform (Fig. 2) and the analytic form of the effective contact resistance is straightforward—$R_c = V_s/I$. However, if the via is sufficiently large the current will be nonuniform and will crowd near the edges of the via as shown in Fig. 3. This "current crowding" effect is a consequence of the current seeking the least resistive path through the structure. We assume that the material and interfaces of the structure have uniform electrical characteristics. Given this assumption the degree of current

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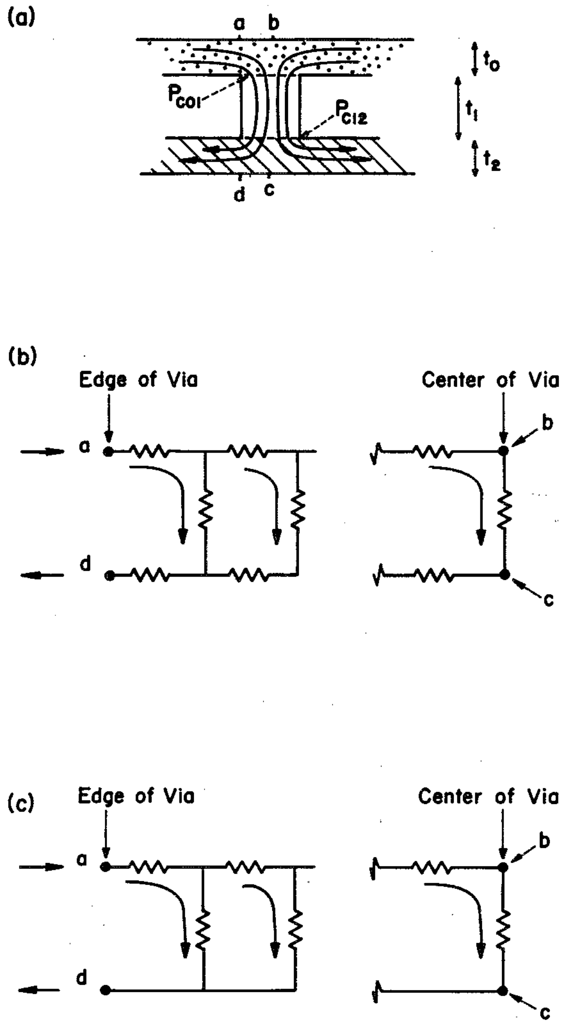

FIG. 1. (a) Cross section of plate/plug/plate contact structure and electrical analog of a distributed resistive network which represents the (b) simplified and (c) general contact structure.

crowding will only be a function of the via dimensions and the specific electrical properties of the material.

## III. SIMPLIFIED STRUCTURE

We begin the analysis by identifying that region of the system which is of primary interest—the region localized to the plug and the thin-films immediately above and below it. It is assumed that the lateral current density in the (top and bottom) plates is uniform up to the edge of the via.

Because of the symmetry of the structure, the problem can be modeled in two dimensions (2-D) having radial ($r$) and vertical ($z$) coordinates. This 2-D problem can be solved by using computer-aided numerical techniques (see Sec. V). However, in order to obtain simple analytic solutions an additional "thin-film" approximation will be imposed. This approximation requires that the majority of the voltage difference along the vertical path from the top of the top plate to the bottom of the bottom plate is due to the interface resistance. This enables the problem to be modeled in one dimension (1-D) using only the radially coordinate "$r$". In order to further simplify the problem we also require that the resistivity of the cylindrical and the bottom plate material be negligible ($R_{sh2} = 0$ and $\rho_1 = 0$) and therefore ignored. Quantitatively these approximations and simplifications can be expressed as

$$P_{c01} \gg R_{sh0}\, t_0^2 \tag{1}$$

where $P_{c01}$ is the specific contact resistivity ($\Omega$ cm$^2$) of the interface. $R_{sh0}$ ($\Omega$) and $t_0$ are, respectively, the sheet resis-

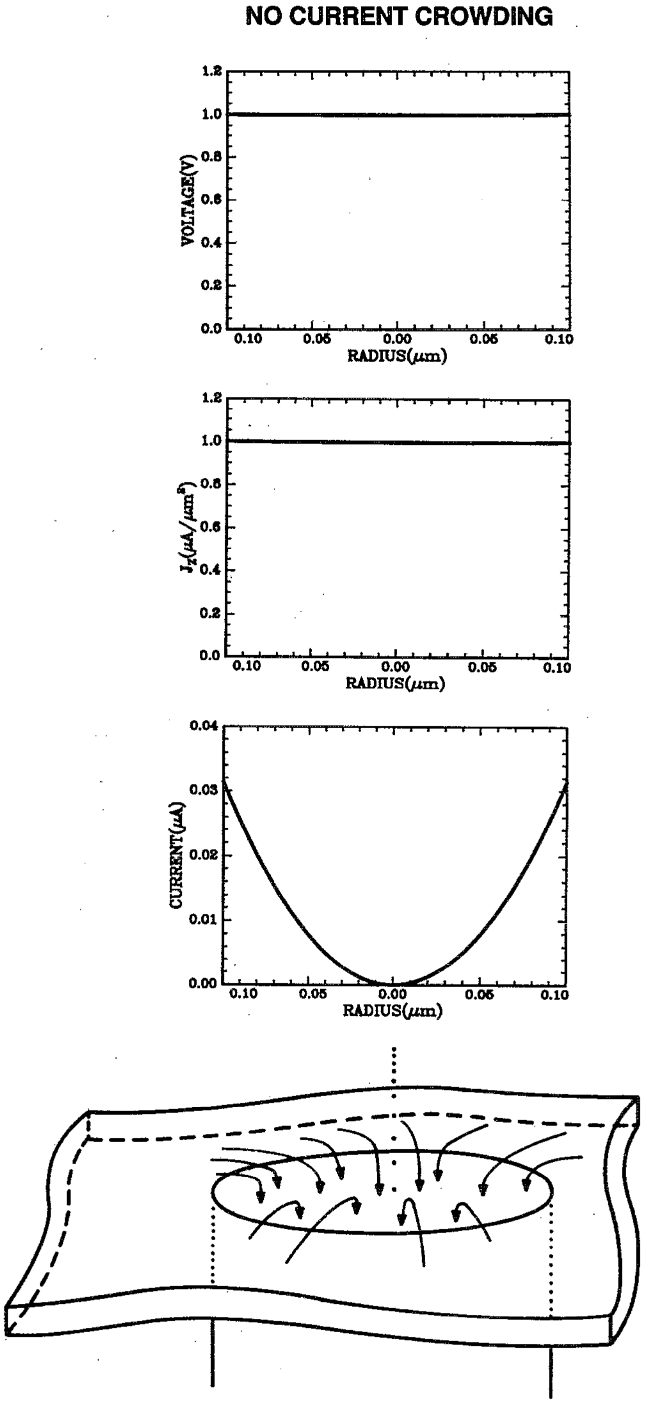

FIG. 2. A cross section distribution profile of the $Jz$, $I$, and $V$ for a contact which shows uniform current density without current crowding.

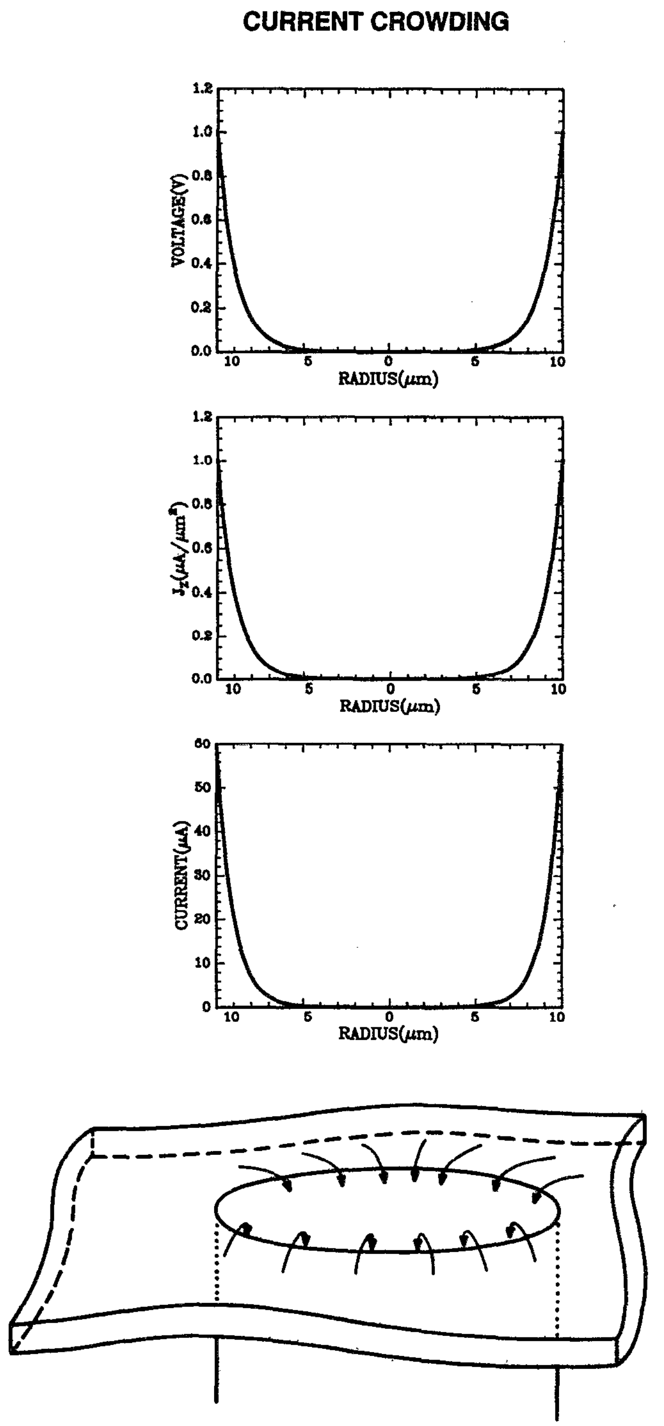

FIG. 3. A cross section distribution profile of the *Jz*, *I*, and *V* for a contact which shows nonuniform current density with current crowding.

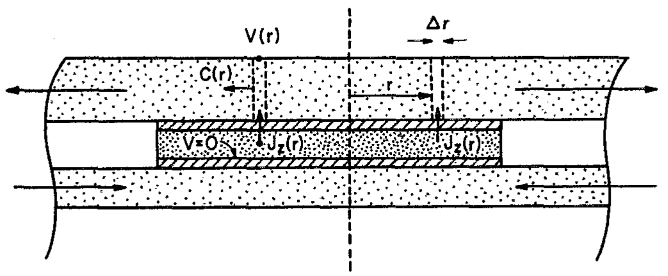

FIG. 4. Cross section diagram of the distribution current ($I=C$) in the contact showing the differential element used in modeling.

tance and the thickness of the top thin film. Given these restrictions the current will have either vertical or horizontal components—no current bending.

Having outlined the problem in this way, it is similar in format to the problem Berger[1] analyzed. The electrical analog of the problem can be represented by a distributed network of resistors as shown in Fig. 1(b).

To obtain an analytic solution, we describe the problem in terms of a differential equation. The differential element is shown in Fig. 4. The vertical current density $J_z(r)$, can be related to the potential $V(r)$ and the interface contact resistivity $P_{c01}$, through the following relation,

$$J_z = V/P_{c01}, \tag{2}$$

where $V$(volts) is the vertical voltage difference between the top and the bottom of the via. The horizontal current $I(r)$ is the current through the annular ring of radius "$r$" as shown in Fig. 4. It is related to the derivative of the voltage $[dV/dr = V'(r)]$ using Ohm's law, by the following expression:

$$I(r) = (2\pi r/R_{sh})V'. \tag{3}$$

Finally, the vertical current density $J_z(r)$ and horizontal current $I(r)$, $[I(r) = 2t_0\pi\, rJ_r(r)]$ are related through the continuity equation

$$I' = (2\pi\, r)J_z, \tag{4}$$

where the derivative of the horizontal current is noted as $dI/dr = I'(r)$. By combining Eqs. (2), (3), and (4), we obtain the differential equation

$$V'' + V'/r - V/L^2 = 0. \tag{5}$$

The term $L^2 = P_{c01}/R_{sh0}$ is constant and is only related to the material and interface properties of the system.

The solution to this equation can be written in terms of a zero order modified Bessel function $I_0(r/L)$:

$$V(r) = V(a)I_0(r/L)/I_0(a/L), \tag{6}$$

where "$a$" is the radius of the cylinder. $V(r)$ represents the voltage profile of the voltage difference between the top and bottom plates as a function of distance "$r$" from the center of the via.

To obtain the current profile $I(r)$, the expression in Eq. (3) is used yielding

$$I(r) = [2\pi\, rV(a)/R_{sh}][I_0'(r/L)/I_0(a/L)] \tag{7}$$

where the derivative of the Bessel function is given by $dI_0/dr = I_0'(r/L) = [1/L]dI_0(q)/dq$ and $q = r/L$.

### A. Current-voltage profiles under various current crowding conditions

The above equations have two independent parameters—"$a$" (via dimensions) and "$L$" (material properties), however it is the ratio $(a/L)$ which quantitatively determines the current and voltage distributions.

If $a/L \ll 1$ (small via radius "$a$", low sheet resistance $R_{sh0}$, and/or high contact resistance $P_{c01}$), there will be negligible current crowding and the current density $J_z$ will be constant as shown in Fig. 2. The voltage difference across the top plate $[\Delta V = I(r = a)\ R_{sh0}/4\pi]$ will be small compared to the average voltage difference $[\approx V(a) - \Delta V]$ across the plate/plug interface (Fig. 2).

However, if the ratio is large, $(a/L \gg 1)$, the degree of current crowding will be significant and most of the current enters and leaves the cylinder near the edge as shown in Fig. 3.

The consequence of current crowding on device performance will depend on the individual situation. For example, in cases where the contacts are part of a nonlinear device (e.g., Schottky diodes) current crowding may produce an apparent nonideal $I$-$V$ curve. This occurs even though the device may have ideal junction characteristics. The following section deals with the effects of current crowding on the contact resistance measurement of ohmic circular contacts.

### B. Contact resistance $R_c$ and contact resistivity $P_c$

There are two parameters, $R_c$ and $P_c$, which are particularly useful in circuit and device modeling. Contact resistance $R_c$ of the via refers to (in this paper) the resistance of the via by itself including only the plug and the thin-film plates immediately above and below. It does not include other series resistance sources, such as the spreading resistance of the plates. $R_c$ is defined in Eq. (8) in terms of the total current $I(r=a)$ delivered to the via and the voltage difference $V(r=a)$ between the top and bottom plates at the edge of the cylinder:

$$R_c \equiv V(r=a)/I(r=a). \tag{8}$$

The value for $R_c$ can be directly determined for a particular via, by measuring current and voltage. However, it may not be a scalable parameter, meaning that the value for $R_c$ may not scale directly with the area of the via. This situation occurs for example, if current crowding is prevalent.

The value for $R_c$ for our simplified structure can be calculated by evaluating Eqs. (6) and (7) at $r=a$ yielding the expression

$$R_c = [R_{sh2}/2\pi a][I_0(a/L)/I_0'(a/L)]. \tag{9}$$

This general expression represents the exact solution relating the contact resistance to contact resistivity. It gives the value for $R_c$ under any conditions of current crowding, but it requires *a priori* knowledge of the material properties of the via and thin film (i.e., $R_{sh0}$ and $P_{c01}$). Values for the sheet resistance $R_{sh0}$ of the thin film can easily be measured, but values for $P_{c01}$ cannot be directly measured.

In most cases it is the value for $P_{c01}$ which is of most interest. Values for $P_{c01}$ can be obtained using Eq. (9), by measuring $R_c$, the sheet resistance and via diameter. This evaluation can be done using iterative numerical techniques. Direct analytic calculations can also be done and this will be discussed in a later section (VI).

We summarize this section, by noting that the parameter $R_c$ can be either measured directly or calculated by using Eq. (9), given values for $R_{sh0}$, $a$, and $P_{c01}$. Alternatively the value for $P_{c01}$ can be obtained by using Eq. (9) if the other quantities $R_{sh0}$, $a$, and $R_c$ are known.

## IV. GENERAL STRUCTURE

In the previous section we analyzed a simplified plate/plug/plate structure where the resistivity of the plug and lower plate material was negligible. In this section we analyze the more general problem where all three materials have significant values of resistance.

In order to simplify the problem we require that the effective "vertical resistance" along any vertical path through the via is due to the combination of the plug resistance, interface contact resistances $P_{c01}$ and $P_{c12}$. As in the previous section, this requirement precludes any current bending within the structure. A quantitative statement of this requirement for the general structure is given as follows:

$$P_c \approx P_{c01} + P_{c12} + \rho_1 t_1 \gg R_{sh0} t_0^2 + R_{sh2} t_2^2. \tag{10}$$

$P_c(\Omega\ \mathrm{cm}^2)$ the total contact resistivity,

$P_{c01}, P_{c12}(\Omega\ \mathrm{cm}^2)$ contact resistivity of the two interfaces,

$R_{sh0}, R_{sh2}(\Omega)$ sheet resistance of the top and bottom layers,

$\rho_1(\Omega\ \mathrm{cm})$ resistivity of the plug material,

$t_0, t_1, t_2(\mathrm{cm})$ thicknesses of the layers.

Shown in Fig. 1(c), is a diagram representing the distributed resistance network of the via system. Our goal is to relate the contact resistance to the other parameters of the system, which is easily accomplished due to the requirements just stated. The contact resistance for the general structure will have the same form as expressed in Eq. (9). The difference between the results will be a matter of substitution since the component resistive factors all add in series. The contact resistance for the general structure is expressed as

$$R_c = [(R_{sh0} + R_{sh2})/2\pi a][I_0(a/L)/I_0'(a/L)]. \tag{11}$$

The term $(R_{sh0} + R_{sh2})$ can be substituted for factor $R_{sh0}$, and $(P_{c01} + P_{c12} + \rho_1 t_1)$ can be substituted for $P_{c01}$. The parameter "$L$" is expressed as

$$L^2 = (P_{c01} + P_{c12} + \rho_1 t_1)/(R_{sh0} + R_{sh2}). \tag{11a}$$

## V. COMPARISON BETWEEN ANALYTIC AND NUMERICAL CALCULATIONS

To demonstrate the accuracy of the analytic solution [Eq. (11)] a comparison is made with a more realistic two-dimensional model of the via contact using computer

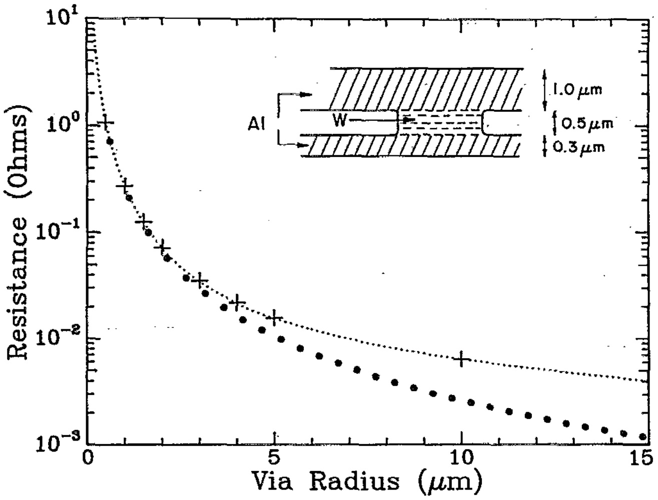

FIG. 5. Comparison of contact resistance vs via radius by 2-D numerical techniques ( + ) and analytical (······) solution from Eqs. (13) and (17). Also shown is a plot (●●●) of $R_c = 0.07/\pi a^2$ where the effects of current crowding are not taken into account and therefore the data is in error.

aided numerical calculations. The comparison was made on an Al/W/Al structure as shown in Fig. 5. The interface contact resistivity of the Al/W and W/Al was $P_c = 0.4$ $\Omega\,\mu\text{m}^2$. For this particular structure, the thin-film conditions [Eq. (10)] are satisfied and therefore it is appropriate to use the analytical solutions. The numerical values for $P_c$ in this case were obtained by numerically solving the Laplacian using a two dimensional mesh. A plot of the current density for two vias of different diameter is shown Fig. 6. A comparison between analytic and 2-D solutions is displayed in Fig. 5 and shows good agreement. The plot also shows the effects of current crowding, as is the case for large vias.

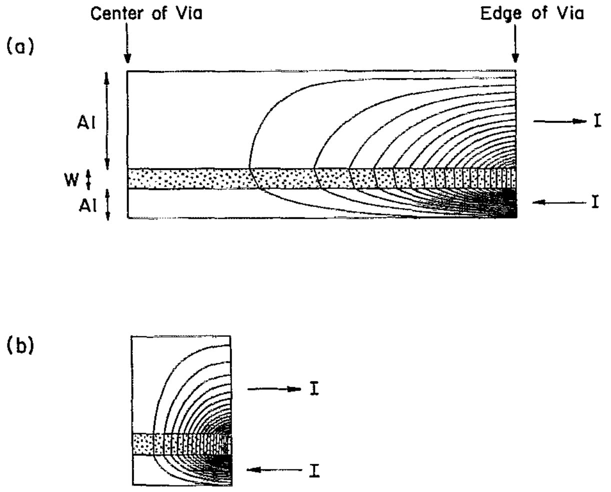

FIG. 6. A cross sectional plot of the current density in (a) 5 $\mu$m radius via (current crowding), and (b) 1 $\mu$m radius via (approximately uniform current density). The data was obtained by 2-D numerical analysis. The Al(1.0 $\mu$m)/W(0.2 $\mu$m)/Al(0.3 $\mu$m) structure had an interface (Al/W) resistivity of $P_c = 0.1\ \Omega\,\mu\text{m}^2$.

## VI. APPROXIMATION FOR ANALYTICAL SOLUTIONS

Equation (11) represents the exact solution for the one-dimensional problem of determining contact resistance at a circular via. However, the Bessel[4] functions (and their derivatives) are cumbersome to manipulate algebraically, especially in the effort to obtain values for $P_c$ which requires iterative evaluations. In this section we develop two simple explicit expressions for $R_c$, which can be used in lieu of Eq. (11).

The hyperbolic Bessel functions ($I_0$) can be expressed using polynomial functions with constant coefficients ($\alpha_i$, $\beta_i$, etc.).[4] For values of $r/L \ll 1$ the function $I_0$ has the form,

$$I_0(r/L) = 1 + \alpha_1(r/L)^2 + \alpha_2(r/L)^4 + \ldots \qquad (12)$$

By using Eqs. (9), (11), and (11a), rearranging terms, and making the necessary approximations, we obtain the first order approximation solutions to $P_c$.

The expression for $P_c$ for small values of $(r/L)$ is given as

$$P_c = F_s R_c \pi a^2, \qquad (13)$$

where

$$F_s = 1 - R_{sh}/(30R_c). \qquad (14)$$

The term $30R_c$ in Eq. (14) was changed from $8\pi R_c$, which was derived from the first order approximation, in order to obtain more accurate values for $P_c$. Expression (13) is applicable over the range of

$$R_{sh}/R_c < 12.0. \qquad (15)$$

For values of $r/L \gg 1$ the hyperbolic Bessel function can be written as

$$I_0(r/L) = (r/L)^{-1/2}\exp(r/L)[1 + \beta_1(r/L)^1 + \beta_2(r/L)^2 + \ldots]. \qquad (16)$$

By truncating the series and making suitable approximations the contact resistivity $P_c$ can be expressed as

$$P_c = (R_c 2\pi a F_B)^2/R_{sh}, \qquad (17)$$

where

$$F_B = 1 - 2.9R_c/R_{sh}. \qquad (18)$$

This solution is applied over the range

$$R_{sh}/R_c > 12.0. \qquad (19)$$

The term $2.9R_c$ in Eq. (18) was changed from $\pi R_c$ (which was derived from the first order approximation), so as to increase the accuracy for $P_c$ over the entire range of values for $R_{sh}/R_c$.

Together these approximate solutions estimate the value for $P_c$ to within $\approx 2\%$ over the entire range of values. To apply these solutions to a particular system, we first measure $R_c$, $R_{sh}$, and "$a$", then decide which solution is applicable [from Eqs. (15) and (19)], and then calculate the value for $P_c$ with either Eq. (13) or Eq. (17).

## VII. DISCUSSION AND CONCLUSIONS

Current crowding problems at circular vias were investigated. Expressions were obtained which give the profile of current and voltage, showing the effects of current crowding. Analytic solutions relating the contact resistance $R_c$ to contact resistivity $P_c$ were also developed. These solutions can be applied to circular via systems regardless of the degree of current crowding. The solutions could be used in designing circuits as well as for obtaining values for $P_c$ when the spreading resistance method[3] for contact resistance is used.

## ACKNOWLEDGMENTS

Cornell work supported in part by Cornell Microscience center of the Semiconductor Research Corporation. We also are grateful for the funding support and cooperation from the IBM General Technology division.

[1] H. H. Berger, J. Electrochem. Soc. **119**, 507 (1972).
[2] W. M. Loh, S. E. Swirhun, E. Crabbe, K. Saraswat, and R. M. Swanson, IEEE Electron Dev. Lett. 6, **441** (1985).
[3] L. H. Allen, E. G. Colgan, R. Young and R. Cook, J. W. Mayer (unpublished).
[4] G. Arfken, *Mathematical Methods for Physicists* (Academic, New York, 1985).